

Section 2.2

Limits: A Numerical and Graphical Approach

(1) Limit Definitions

- (A) Two-Sided Limits
- (B) One-Sided Limits

(2) Estimating Limits

(3) Zeno's Paradox

▶ [Link](#)

Limits

One-Sided Limit

$$\lim_{x \rightarrow c^-} f(x) = L$$

The left limit of f is L if the y -values converge to L as x approaches c through values **less than** c .

$$\lim_{x \rightarrow c^+} f(x) = L$$

The right limit of f is L if the y -values converge to L as x approaches c through values **greater than** c .

Two-Sided Limit

$$\lim_{x \rightarrow c} f(x) = L$$

The two-sided limit of f is L **ONLY IF** both one-sided limits equal L .

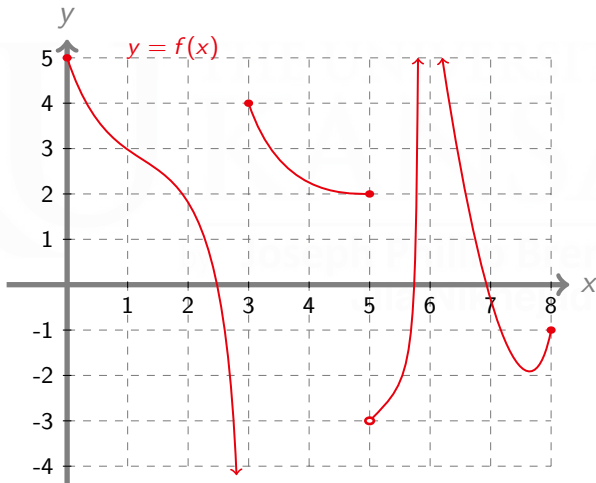
Identifying Limits on a Graph - Example 1

Discuss the limits and actual values on the following graph:



Identifying Limits on a Graph - Example 2

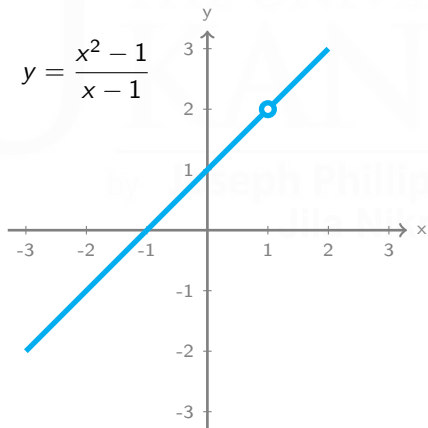
Discuss the limits and actual values on the following graph:



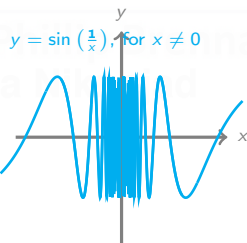
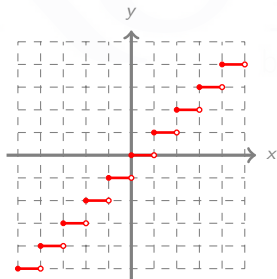
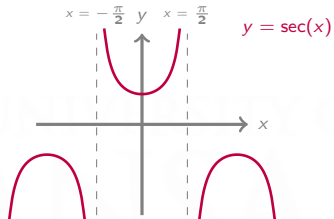
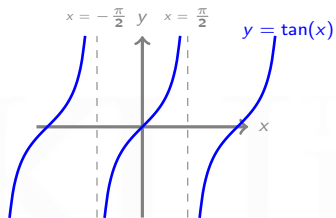
Estimating Limits

Let $f(x) = \frac{x^2 - 1}{x - 1}$. What is the limit of $f(x)$ as $x \rightarrow 1$?

x	0.9	0.99	0.999	0.99999	1	1.000001	1.001	1.01	1.1
y	1.9	1.99	1.999	1.99999	DNE	2.000001	2.001	2.01	2.1



Limits That Don't Exist



Zeno's Paradox



Zeno walks towards a wall by walking exactly half the distance remaining between him and the wall. Will he ever reach the wall?

Limits and Actual Values are **INDEPENDENT** of Each Other

We will be interested in certain situations where the limit and actual value at a point exist and are equal. This concept is called **continuity**; more in section 2.4!

- Hair Cuts: What are the one-sided limits at the instant of cutting?
- Meter Rates: Electric company increases charges and institutes fees at a certain point, what are the one sided limits at that point?

The Unexpected: Truman is sailing his boat triumphantly away from his home when he suddenly hits the transparent wall enclosing his world... only then does he understand that his world was artificial.



Niccol, A., & Weir, P. (1999). The Truman

Sewer Hole

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